A Unified Diagnostic System for Uncertainty Analysis of Land Carbon Cycle Models

Yiqi Luo

Center for Ecosystem Science and Society, Northern Arizona University, AZ, USA

ECOLAB OF DR. YIQI LUO

Yiqi.Luo@nau.edu

http://www2.nau.edu/luo-lab/?home

RUBISCO, Feb 16, 2018

NORTHERN ARIZONA UNIVERSITY

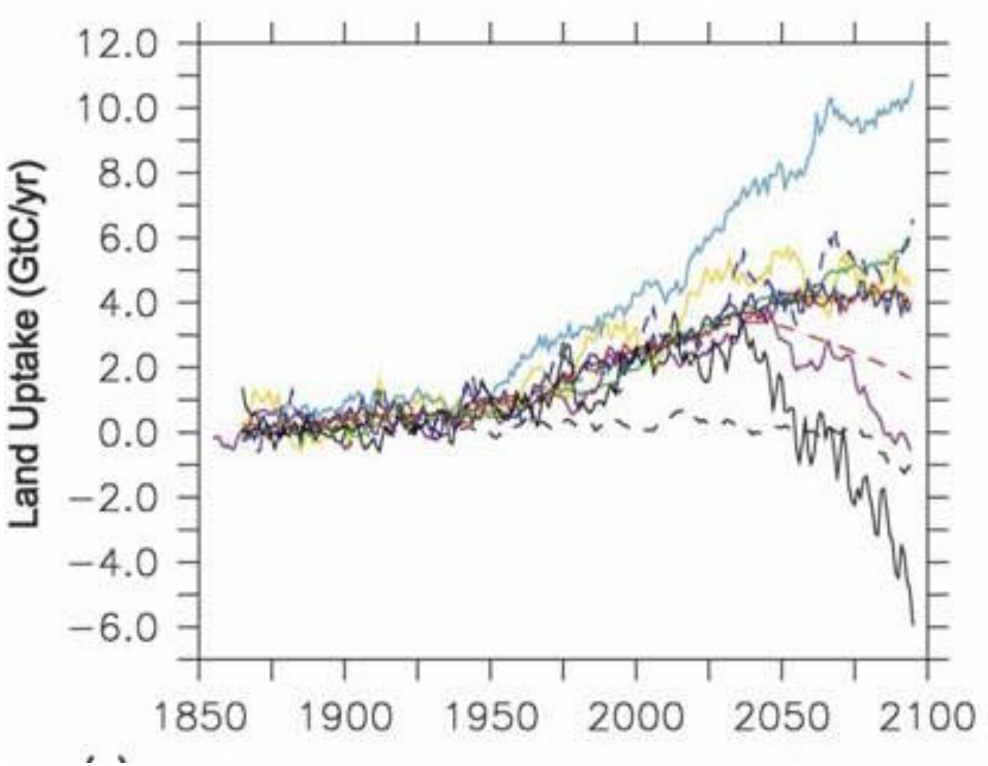
Unified Diagnostic System
Or 1-3-5 scheme

- One (1) formulae unifies all land C cycle models
- One 3-D space to evaluate all model outputs
- Five (5) Traceable components to pinpoint uncertainty sources

Luo et al. AGU talk 2017

Background

Uncertainty in land carbon cycle modeling



Friedlingstein et al. 2006

- Models behave so differently;
- Uncertainty has been documented in almost all model intercomparison projects (MIPs);
- Uncertainty becomes larger instead of smaller as we incorporate more processes into models
- We become more confused with uncertainty as we invest more time to address this issue.

Modeling conundrum

Increasing detail in process representation in models, and the simulations they produce, hinders our understanding of holistic system behavior

Conundrum in climate modeling

High degree of complexity and sophistication of model implementations hinders understanding of general patterns of atmospheric circulation and climate dynamics.

Matrix approach

Matrix representation of land carbon cycle provides a general framework for the qualitative understanding of models without compromising detail in process representation

Sierra et al. under review

Unified Diagnostic System
Or 1-3-5 scheme

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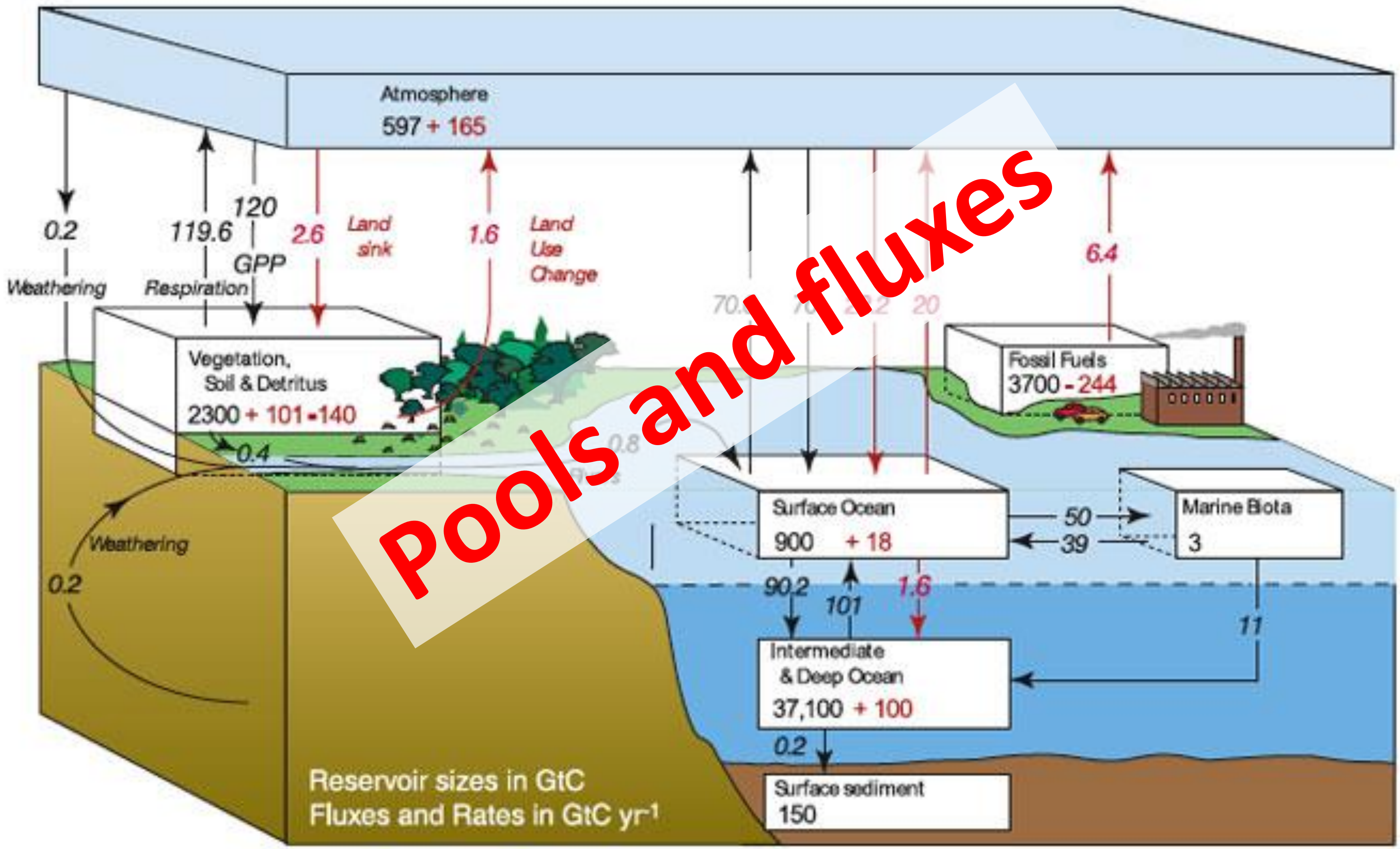
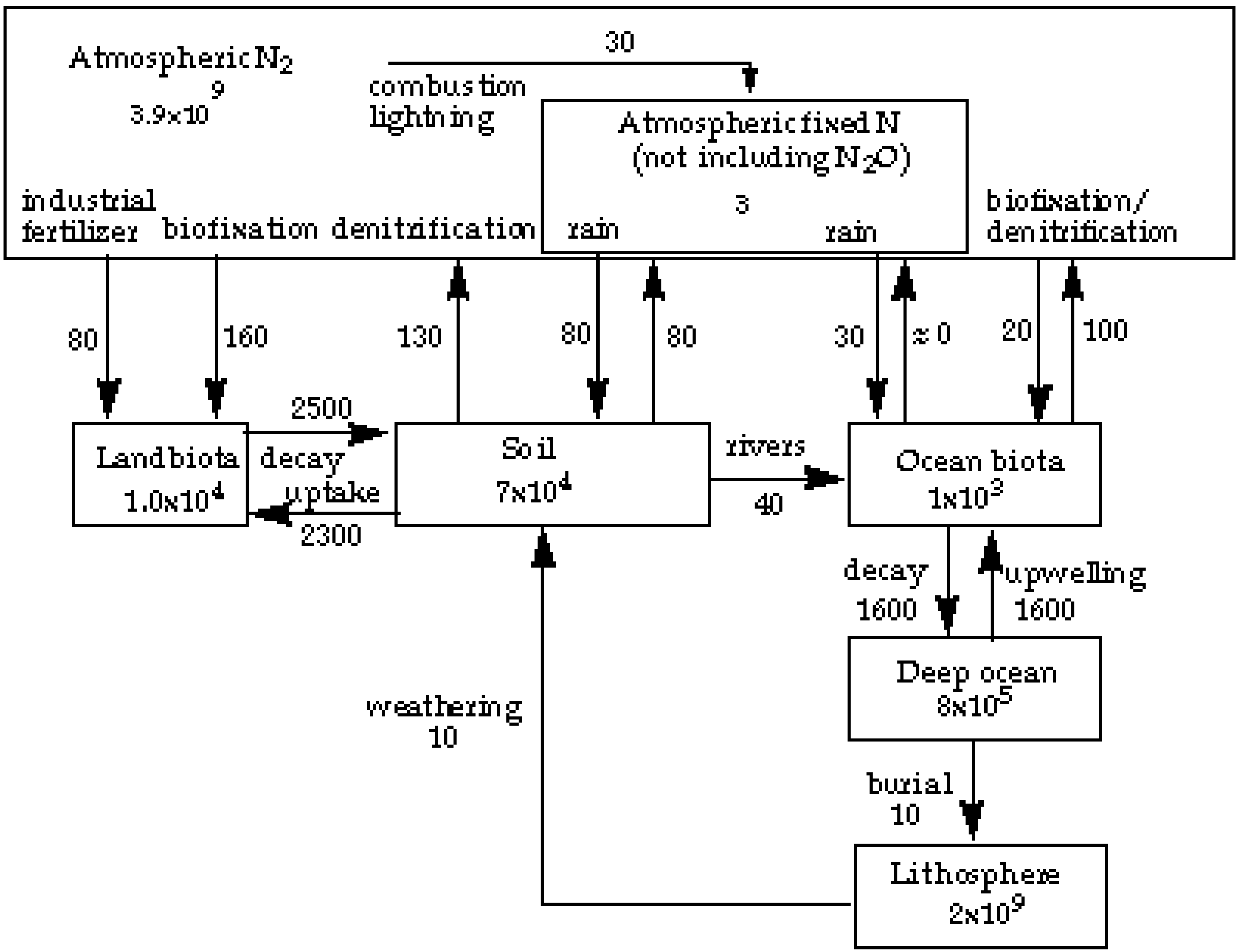
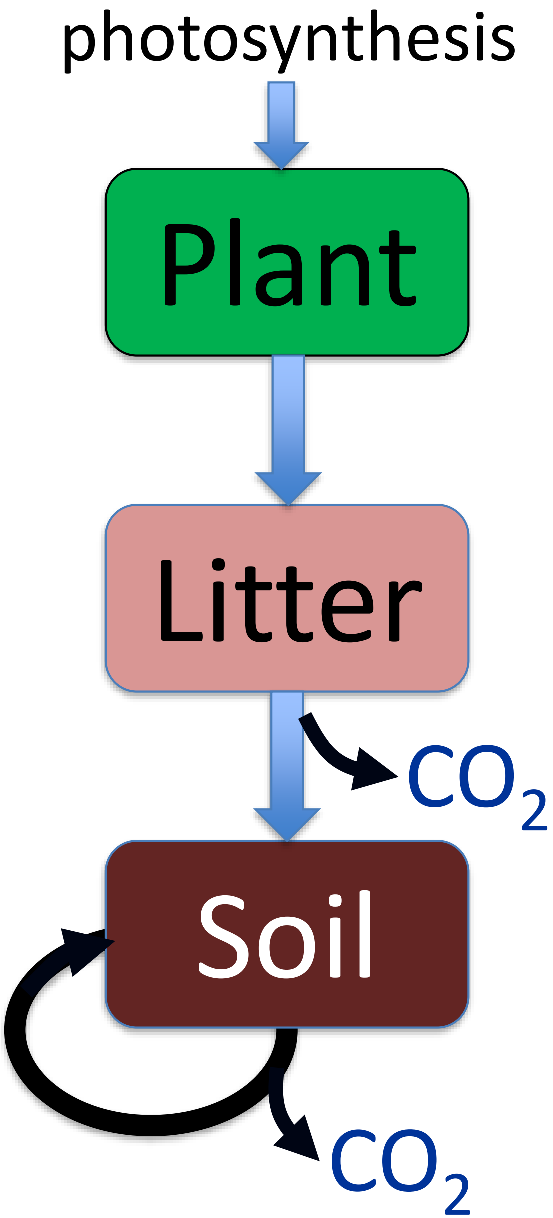
Global carbon cycle



Box-arrow model to track pools and fluxes



Three-pool model



litterfall

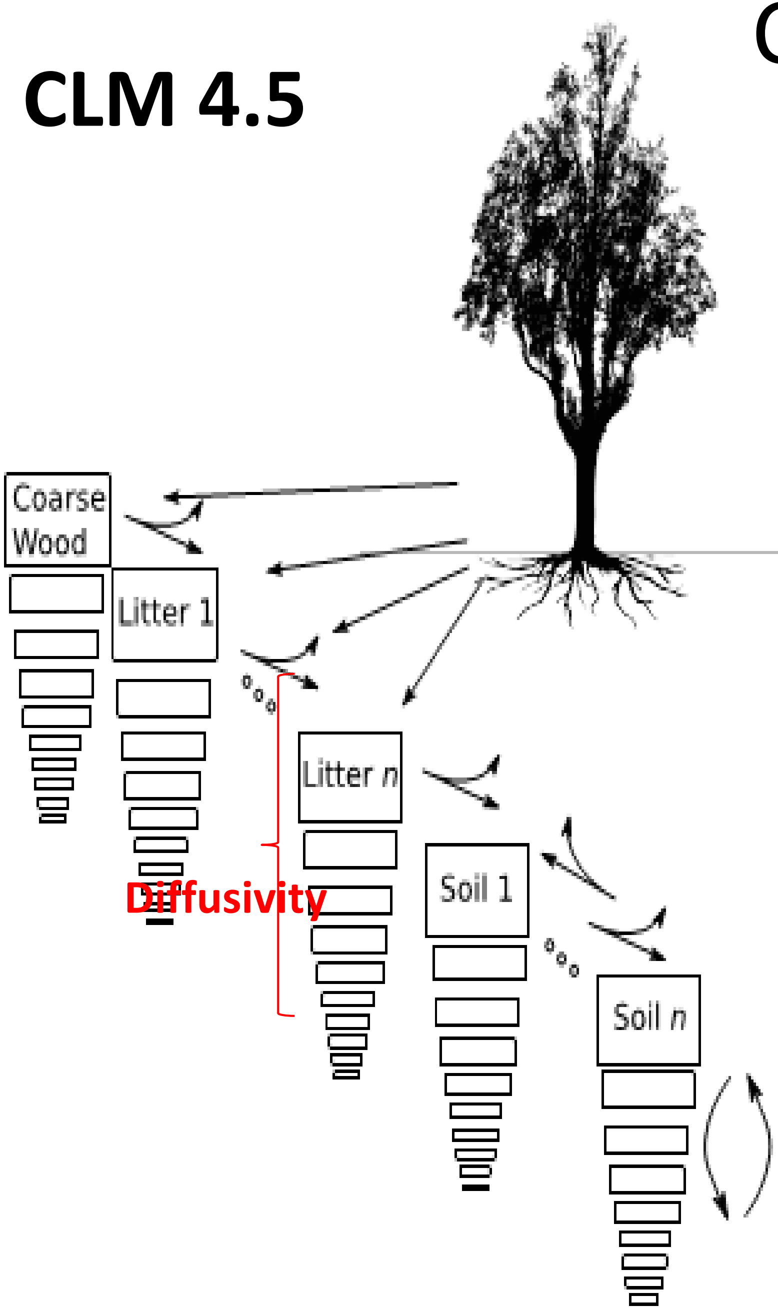
Decomposition

Mineralization

Stabilization

Complex model

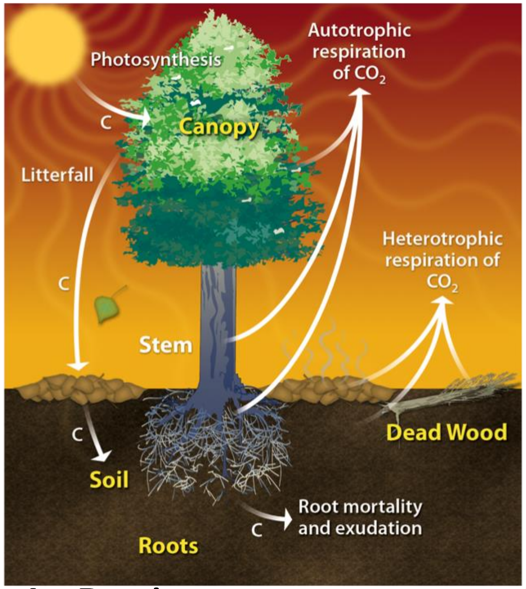
CLM 4.5



Plant pools (306)
18 per vegetation type
17 vegetation types

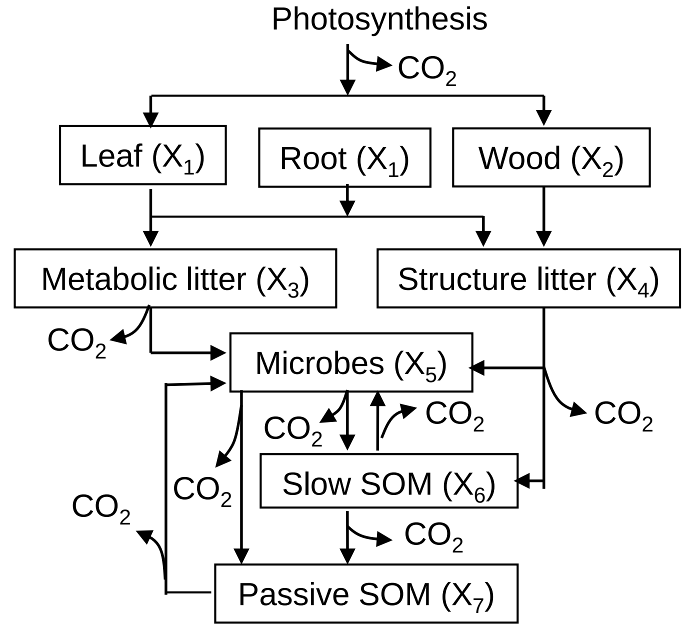
Soil pools (70)
7 per soil layer
10 layers

376 carbon pools
378 nitrogen pools

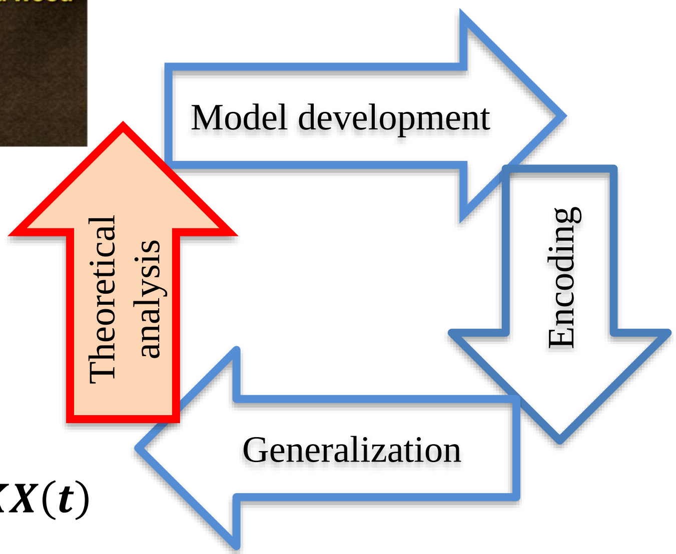


A: Basic processes

One formula to unify all land carbon cycle models



B: Shared model structure



D: General model

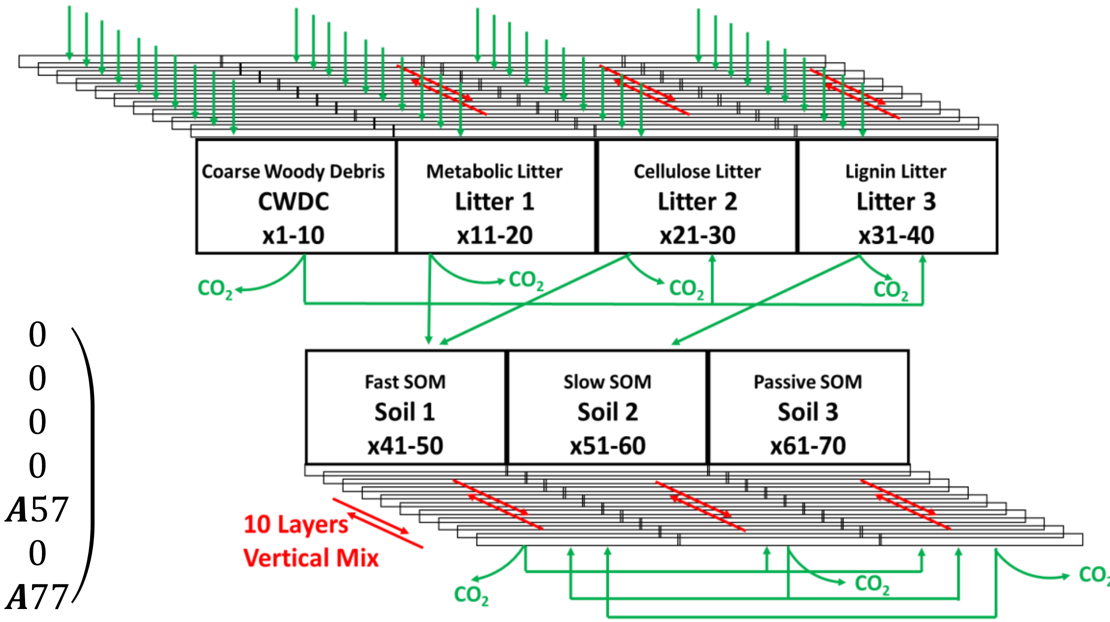
$$\frac{dX(t)}{dt} = BI(t) - A\xi(t)KX(t)$$

C: Similar algorithm

$$
\begin{cases}
Plant \begin{cases}
dX_1(t)/dt = b_1U(t) - \xi c_1 X_1(t) \\
dX_2(t)/dt = b_2U(t) - \xi c_2 X_2(t) \\
dX_3(t)/dt = b_3U(t) - \xi c_3 X_3(t)
\end{cases} \\
Litter \begin{cases}
dX_4(t)/dt = \xi[c_1 a_{41} x_1(t) + c_3 a_{43} x_3(t) - c_4 X_4(t)] \\
dX_5(t)/dt = \xi[c_1 a_{51} x_1(t) + c_2 x_2(t) + c_3 a_{53} x_3(t) - c_5 X_5(t)]
\end{cases} \\
SOM \begin{cases}
dX_6(t)/dt = \xi[c_4 a_{64} x_4(t) + c_5 a_{65} x_5(t) + c_7 a_{67} x_7(t) + c_8 a_{68} x_8(t) - c_6 X_6(t)] \\
dX_7(t)/dt = \xi[c_5 a_{75} x_5(t) + c_6 a_{76} x_6(t) - c_7 X_7(t)] \\
dX_8(t)/dt = \xi[c_6 a_{86} x_6(t) + c_7 a_{87} x_7(t) - c_8 X_8(t)]
\end{cases}
\end{cases}
$$

Luo et al. 2001
Luo et al. 2003 GBC
Luo and Weng 2011 TREE
Luo et al. 2012
Luo et al. 2015
Luo et a. 2017

Matrix equation of CLM4.5



$$\frac{d\boldsymbol{X}(\boldsymbol{t})}{d\boldsymbol{t}} = \boldsymbol{B}(\boldsymbol{t})\boldsymbol{I}(\boldsymbol{t}) - \boldsymbol{A}\boldsymbol{\xi}(\boldsymbol{t})\boldsymbol{K}\boldsymbol{X}(\boldsymbol{t}) - \boldsymbol{V}(\boldsymbol{t})\boldsymbol{X}(\boldsymbol{t})$$

$$X(t) = (X_1(t), X_2(t), X_3(t), \dots, X_{70}(t))^T$$

$$\mathbf{A} = \begin{pmatrix} \boldsymbol{A}11 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{A}22 & 0 & 0 & 0 & 0 & 0 \\ \boldsymbol{A}31 & 0 & \boldsymbol{A}33 & 0 & 0 & 0 & 0 \\ \boldsymbol{A}41 & 0 & 0 & \boldsymbol{A}44 & 0 & 0 & 0 \\ 0 & \boldsymbol{A}52 & \boldsymbol{A}53 & 0 & \boldsymbol{A}55 & \boldsymbol{A}56 & \boldsymbol{A}57 \\ 0 & 0 & 0 & \boldsymbol{A}64 & \boldsymbol{A}65 & \boldsymbol{A}66 & 0 \\ 0 & 0 & 0 & 0 & \boldsymbol{A}75 & \boldsymbol{A}76 & \boldsymbol{A}77 \end{pmatrix}$$

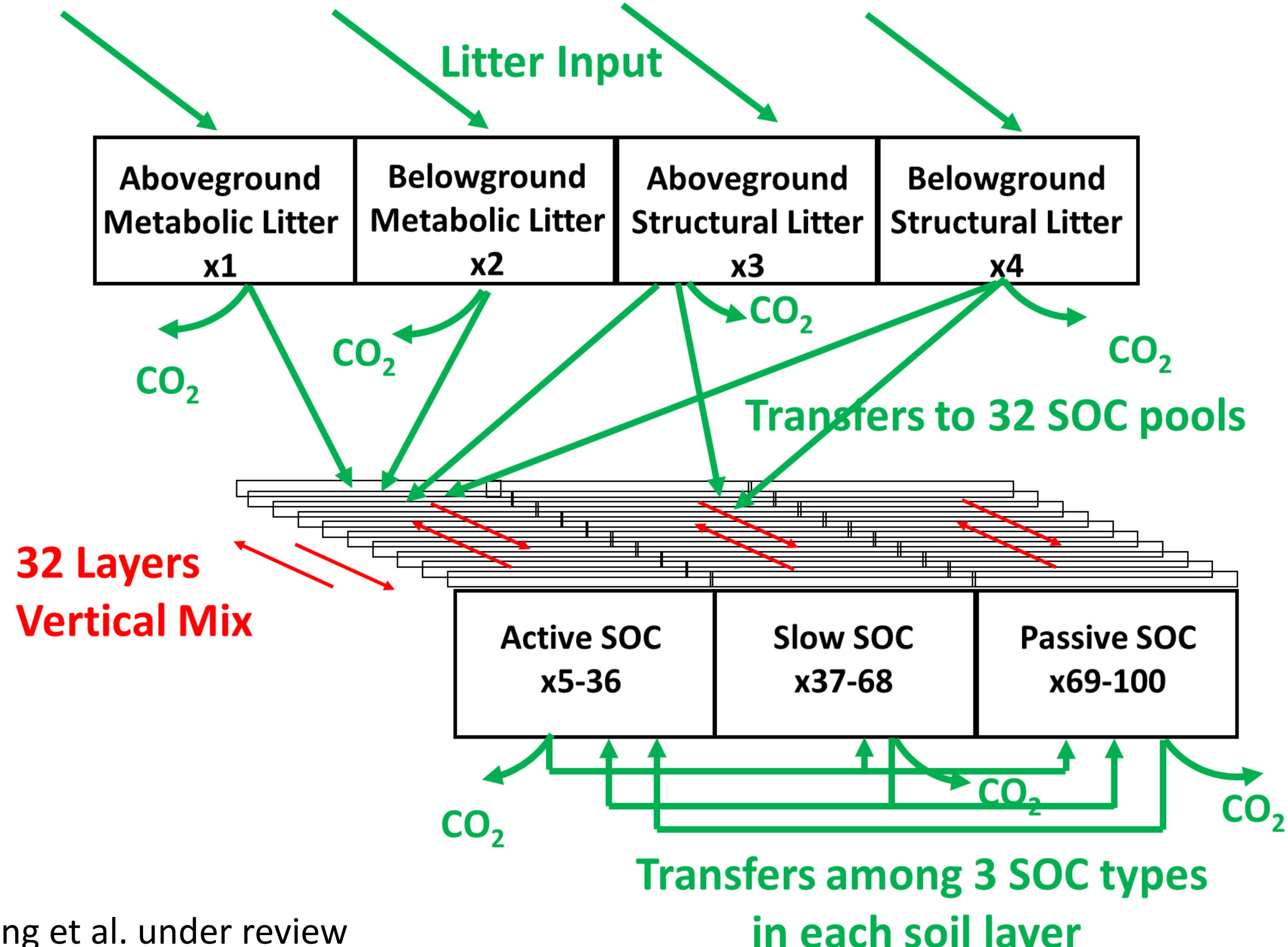
$$\mathbf{A}_{31} = diag(-f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31})$$

$$\mathbf{V}(\mathbf{t}) = \begin{pmatrix} \boldsymbol{V}11 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{V}22(\boldsymbol{t}) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{V}33(\boldsymbol{t}) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \boldsymbol{V}44(\boldsymbol{t}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \boldsymbol{V}55(\boldsymbol{t}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \boldsymbol{V}66(\boldsymbol{t}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \boldsymbol{V}77(\boldsymbol{t}) \end{pmatrix}$$

Huang et al. 2018
Global Change Biology

$$\boldsymbol{V}22 = \boldsymbol{diag}(\boldsymbol{z}_1, \boldsymbol{z}_2, \dots, \boldsymbol{z}_{10})^{-1} \begin{pmatrix} \boldsymbol{g}_1 & -\boldsymbol{g}_1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -\boldsymbol{h}_2 & \boldsymbol{h}_2 + \boldsymbol{g}_2 & -\boldsymbol{g}_2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -\boldsymbol{h}_3 & \boldsymbol{h}_3 + \boldsymbol{g}_3 & -\boldsymbol{g}_3 & \cdots & 0 & 0 & 0 \\ 0 & 0 & -\boldsymbol{h}_4 & \boldsymbol{h}_4 + \boldsymbol{g}_4 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \boldsymbol{h}_8 + \boldsymbol{g}_8 & -\boldsymbol{g}_8 & 0 \\ 0 & 0 & 0 & 0 & \cdots & -\boldsymbol{h}_9 & \boldsymbol{h}_9 + \boldsymbol{g}_9 & -\boldsymbol{g}_9 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -\boldsymbol{h}_{10} & \boldsymbol{h}_{10} \end{pmatrix}$$

ORCHIDEE matrix model



Huang et al. under review
JAMES

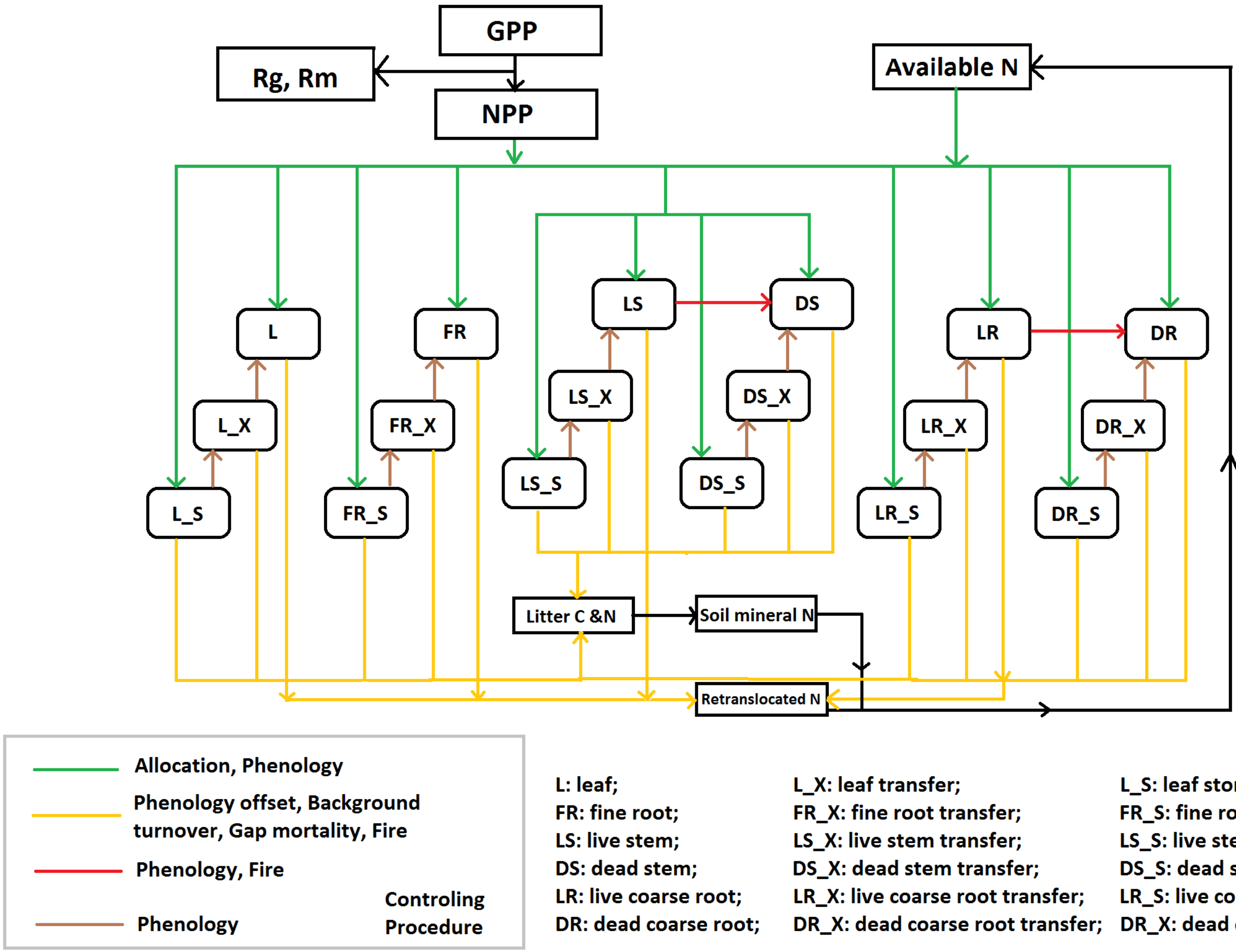
General equation for C and N model

$$\begin{cases} \frac{d}{dt}X(t) = A_C\xi(t)K_C X(t) + u(N,t)B \\ \frac{d}{dt}N(t) = A_N\xi(t)K_N N(t) + k_u F\prod \end{cases}$$

$X(t=0)=X_0$

$N(t=0)=N_0$

CLM vegetation C&N: phenology, fire etc.



L: leaf;	L_X: leaf transfer;	L_S: leaf storage
FR: fine root;	FR_X: fine root transfer;	FR_S: fine root storage
LS: live stem;	LS_X: live stem transfer;	LS_S: live stem storage
DS: dead stem;	DS_X: dead stem transfer;	DS_S: dead stem storage
LR: live coarse root;	LR_X: live coarse root transfer;	LR_S: live coarse root storage
DR: dead coarse root;	DR_X: dead coarse root transfer;	DR_X: dead coarse root storage

Matrix equation of vegetation C&N dynamics

$$\frac{d}{dt}\boldsymbol{X}(t) = (\boldsymbol{A}_{ph}(t)\boldsymbol{K}_{ph}(t) + \boldsymbol{A}_{gm}(t)\boldsymbol{K}_{gm}(t) + \boldsymbol{A}_{fi}(t)\boldsymbol{K}_{fi}(t))\boldsymbol{X}(t) + \boldsymbol{B}(t)\boldsymbol{F}(t)$$

C transfer of phenology

C transfer of gap mortality

C transfer of fire

pool state

input

C turnover of phenology

C turnover of gap mortality

C turnover of fire

allocation

Matrix equation of soil C&N dynamics

$$\frac{d}{dt}\boldsymbol{X}(\boldsymbol{t}) = \left(\boldsymbol{A}\boldsymbol{\xi}(\boldsymbol{t})\boldsymbol{K} - \boldsymbol{V}(\boldsymbol{t}) - \boldsymbol{V}_{\boldsymbol{f}}(\boldsymbol{t})\right)\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}(\boldsymbol{t})\boldsymbol{I}(\boldsymbol{t})$$

C&N pools

Transfer matrix

Scalar

Tridiagonal matrix (diffusion and advection)

allocation

Decomposition rate

Tridiagonal matrix (fire)

input

Diagnostic variables related to C storage Capacity (X_C) and C storage potential (X_P)

$$X_C = -(A\xi K)^{-1}BI$$
$$X_P = X_C - X$$

Luo et al. 2017

ξ: Environmental scalar
A: Carbon transfer coefficient
K: Turnover rate
B: Partitioning coefficients for influx
I: Influx
X: state variable of C storage

Add 100 variables: 36 Vegetation C output variables, 36 Vegetation N output variables (18 vegetation pools), 14 Soil C variables and 14 Soil N variables (7 soil pools) for both capacity and potential.

5. Hierarchical models

Vertical profile

$$\frac{d\boldsymbol{X}(\boldsymbol{t})}{d\boldsymbol{t}} = (\boldsymbol{A}\boldsymbol{\xi}(\boldsymbol{t})\boldsymbol{K} + \boldsymbol{V}(\boldsymbol{t}))\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}(\boldsymbol{t})u(\boldsymbol{t})$$

$$\frac{dX(t)}{dt} = A\xi(t)KX(t) + Bu(t)$$

C transfer of phenology

C transfer of gap mortality

C transfer of fire

pool state

input

$$\frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{X}(\boldsymbol{t}) = (\boldsymbol{A}_{\boldsymbol{ph}}(\boldsymbol{t})\boldsymbol{K}_{\boldsymbol{ph}}(\boldsymbol{t}) + \boldsymbol{A}_{\boldsymbol{gm}}(\boldsymbol{t})\boldsymbol{K}_{\boldsymbol{gm}}(\boldsymbol{t}) + \boldsymbol{A}_{\boldsymbol{fi}}(\boldsymbol{t})\boldsymbol{K}_{\boldsymbol{fi}}(\boldsymbol{t}))\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}(\mathbf{t})\boldsymbol{F}(\boldsymbol{t})$$

C turnover of phenology

C turnover of gap mortality

C turnover of fire

allocation

Vegetation dynamics

Developing models at different levels of complexity under one overarching theory

General representation

	t-dependence	
$\boldsymbol{x}$-dependence	Autonomous	Non-autonomous
Linear	$\boldsymbol{u} + \mathbf{B} \cdot \boldsymbol{x}(t)$	$\boldsymbol{u}(t) + \mathbf{B}(t) \cdot \boldsymbol{x}(t)$
Nonlinear	$\boldsymbol{u}(\boldsymbol{x}) + \mathbf{B}(\boldsymbol{x}) \cdot \boldsymbol{x}(t)$	$\boldsymbol{u}(\boldsymbol{x}, t) + \mathbf{B}(\boldsymbol{x}, t) \cdot \boldsymbol{x}(t)$

Sierra et al. under review

General equation for biogeochemical models

Matrix models

1. CLM 3.5
2. CLM4.0
3. CLM4.5
4. CLM5.0
5. CABLE
6. LPJ-GUESS
7. ORCHIDEE
8. BEPS
9. TECO

In progress

1. JULES
2. LM3V-N

10 more models to participate in the summer training course

10 nonlinear microbial models by Carlos Sierra

Unified Diagnostic System
Or 1-3-5 scheme

- One (1) formulae unifies all land C cycle models
- One 3-D space to evaluate all model outputs
- Five (5) Traceable components to pinpoint uncertainty sources

Major issues

$$\begin{cases} \dfrac{dX(t)}{dt} = Ax(t)CX(t) + BU(t) \\ X(t=0) = X_0 \end{cases}$$

If the carbon cycle mathematically is an extremely simple system,

- How can it account for complex phenomena observed in the real world?

Investigative Workshop in 2012

Jim Cushing: Nonautonomous system

Nonautonomous system

A dynamical system with its input and parameters being time dependent

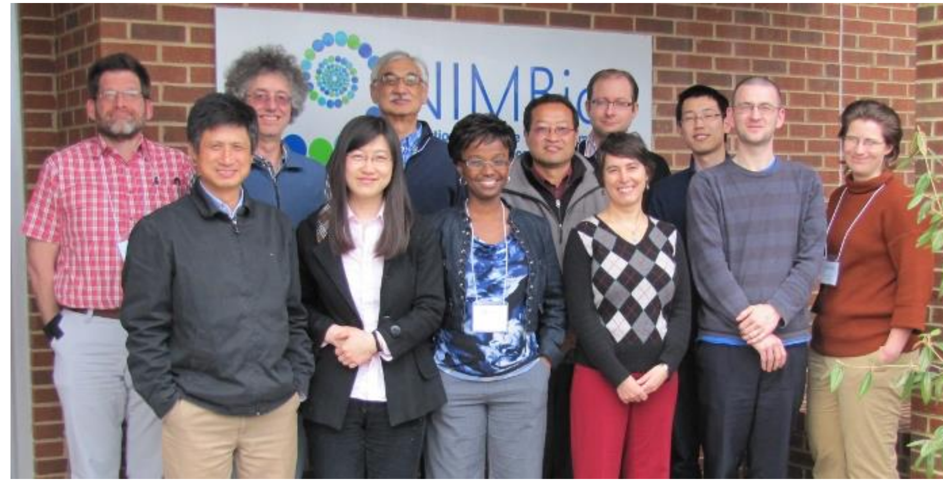
$$\begin{cases} \dfrac{dX(t)}{dt} = Ax(t)CX(t) + BU(t) \\ X(t=0) = X_0 \end{cases}$$

$U(t)$ is input, which is time dependent

Parameters $x(t)$ and $B(t)$ are time dependent

NIMBioS
National Institute for Mathematical
and Biological Synthesis

Working group



Carbon cycle dynamics

$$\frac{dX(t)}{dt} = BI(t) - A\xi(t)KX(t)$$

$$X(t) = (A\xi(t)K)^{-1}Bu(t) - (A\xi(t)K)^{-1}X'(t)$$

$$X(t) = t_E(t)NPP(t) - X_p(t)$$

Transient dynamics

Residence time

Production

Potential

3D

$$X_{ss}(t) = \tau_E(t)NPP(t)$$

Steady state

Luo et al. 2017, *Biogeosciences*

Predictability

<u>External forcing</u> | <u>System equation</u> | <u>Response</u>

External forcing

- Periodic climate (e.g., seasonal)
- Disturbance event (e.g., fire)
- Climate change (e.g., rising CO_2)
- Disturbance regime
- Ecosystem state change (e.g., tipping point)

System equation

$$\begin{cases} \dfrac{dX(t)}{dt} = x(t)ACX(t) + bU(t) \\ X(0) = X_0 \end{cases}$$

Response

- Periodicity
- Pulse-recovery
- Gradual change
- disequilibrium
- Abrupt change

Given one type of forcing, we anticipate a highly predictable pattern of response

Luo et al. 2015 GCB

Review

Dynamic disequilibrium of the terrestrial carbon cycle under global change

Yiqi Luo and Ensheng Weng

Department of Botany and Microbiology, University of Oklahoma, OK 73019, USA

$$\frac{dX(t)}{dt} = BI(t) - A\xi(t)KX(t)$$

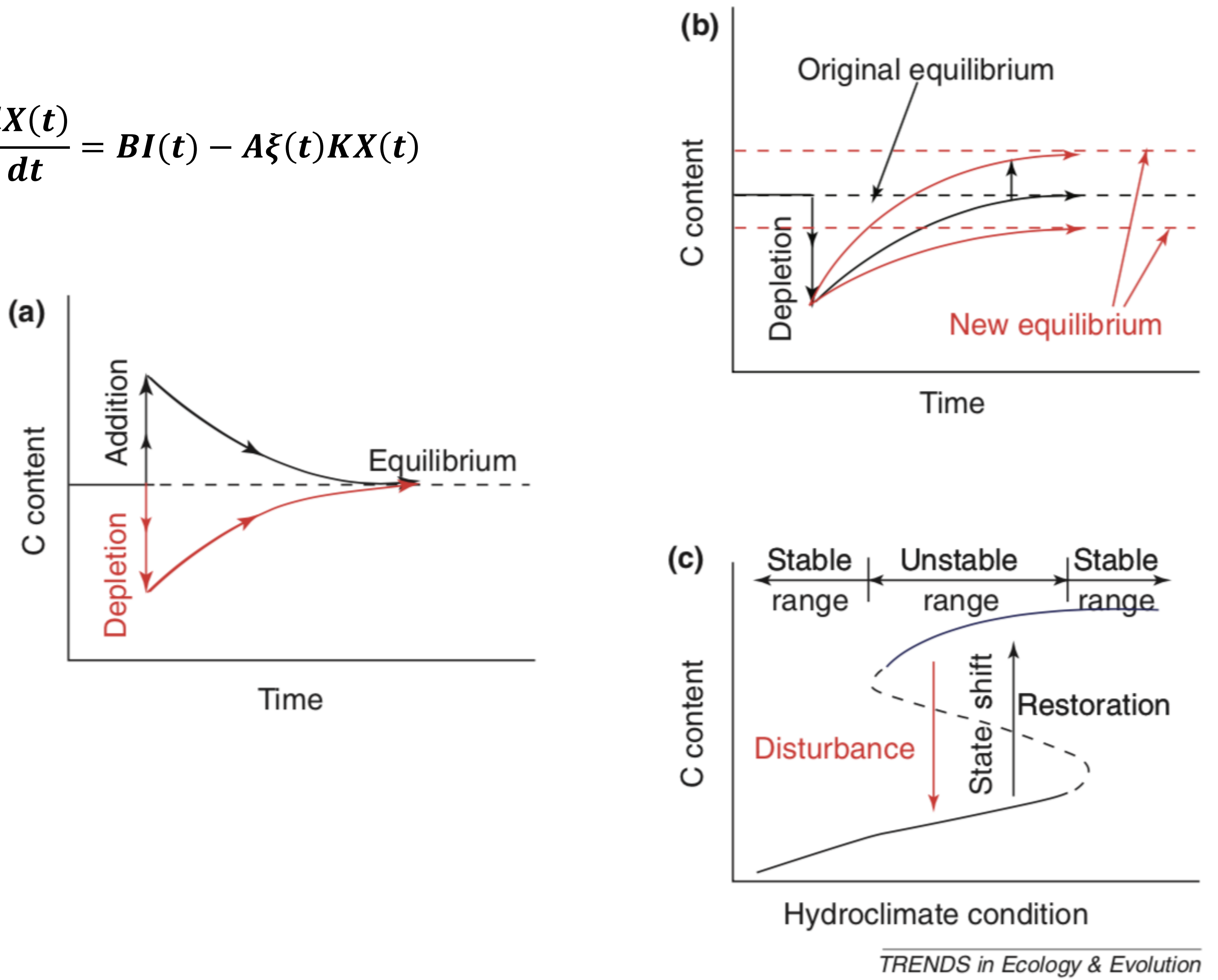


Table 1. Applications of the dynamic disequilibrium concept to assess properties of C sink dynamics in five cases

Case	Equilibrium	Disequilibrium	Methods of quantification	Note
Ecosystem over 1 day and 1 year	Annual averages of C influx and efflux are balanced unless the ecosystem is at disequilibrium owing to disturbance or global change	Diel and seasonal imbalances of C influx and efflux are driven by cyclic environmental change	Diel and seasonal imbalances of C influx and efflux can generally be simulated successfully by models without changes in parameterization	No need to apply the dynamic disequilibrium concept for understanding diel and seasonal dynamics of the C cycle
Global change	An original equilibrium can be defined at a reference condition (e.g. pre-industrial [CO_2]) and a new equilibrium at the given set of changed conditions	Dynamic disequilibrium occurs as the C cycle shifts from the original to a new equilibrium. Global change factors gradually alter over time, leading to continuous dynamic disequilibrium	Direct effects of global change on the C cycle can be modeled via environmental scalars to estimate dynamic disequilibrium explicitly	Dynamic disequilibrium diminishes with acclimation and adaptation, but amplifies with changes in ecosystem structure to new states of the C cycle
Ecosystem within one disturbance–recovery episode	C cycle is at equilibrium if the ecosystem fully recovers after a disturbance. The equilibrium C storage equals the product of C influx and residence time	C cycle is at dynamic disequilibrium and an ecosystem sequesters or releases C before the ecosystem fully recovers to the equilibrium level	C sequestration or release under dynamic disequilibrium can be fully quantified by three sets of parameters related to C influx, residence time and initial pool size	Data assimilation and other techniques are needed to estimate the three sets of parameters simultaneously
Regions with multiple disturbances over time	C cycle is at dynamic equilibrium in a region when the disturbance regime does not shift (i.e. is stationary). The realizable C storage under a stationary regime is smaller than that at the equilibrium level (Figure 2d–f, main text)	C cycle is at dynamic disequilibrium and the region sequesters or releases C when the disturbance regime in the region shifts (i.e. is non-stationary)	Disturbance regime shifts can be characterized by a joint probability distribution of disturbance frequency and severity over space and time. The joint distribution can be combined with C cycle models to estimate regional C sink dynamics over time	Single disturbance events offer no information on regional C sequestration. Probability distribution can be used for prognostic C modeling by generating stochastic forcings of disturbance
Multiple states	C cycle can be at equilibrium at the original and alternative states	Dynamic disequilibrium occurs as an ecosystem changes from the original to alternative states	State changes usually result from changed ecosystem structures to require changes in structures and parameters of C models	State changes can be the major mechanisms for instability of future terrestrial C storage

Carbon cycle dynamics

$$\frac{dX(t)}{dt} = BI(t) - A\xi(t)KX(t)$$

$$X(t) = t_E(t)NPP(t) - X_p(t)$$

Transient dynamics

Residence time

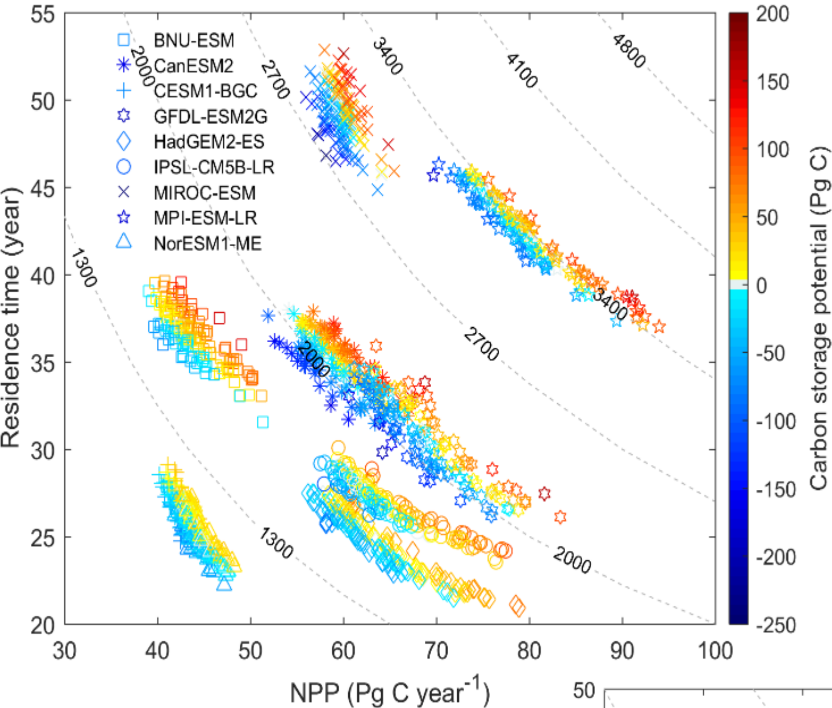
Production

Potential

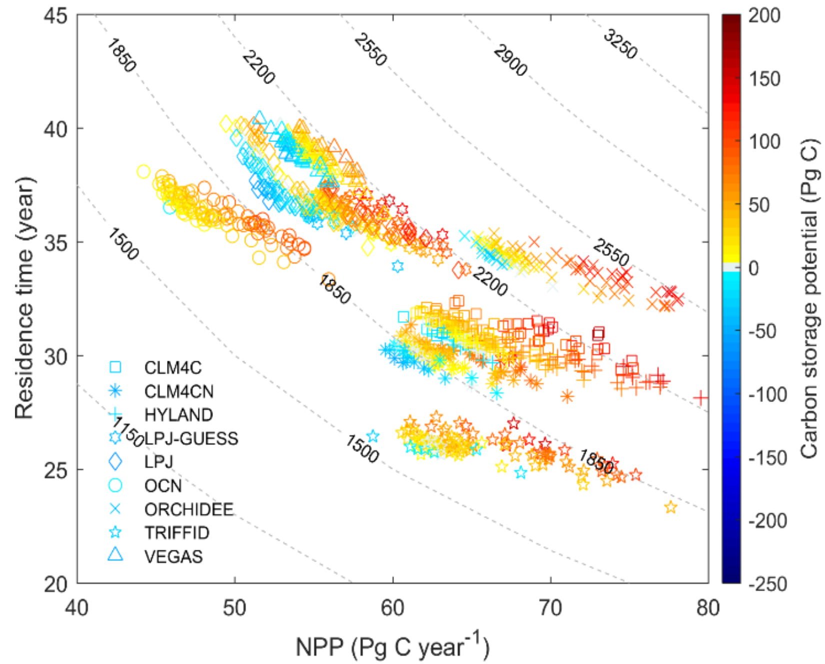
3D

Luo et al. 2017, *Biogeosciences*

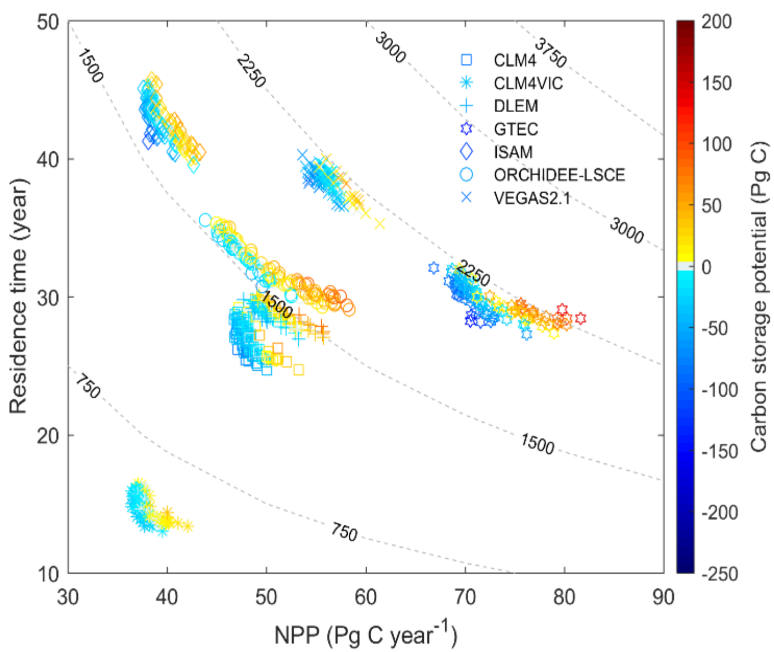
CMIP5



TRENDY



MsTMIP



Zhou et al. 2018

J Climate

Unified Diagnostic System
Or 1-3-5 scheme

- One (1) formulae unifies all land C cycle models
- One 3-D space to evaluate all model outputs
- Five (5) Traceable components to pinpoint uncertainty sources

Carbon cycle dynamics

$$\frac{dX(t)}{dt} = BI(t) - A\xi(t)KX(t)$$

NPP

Scalar

Plant allocation   Microbial CUE   Decomposition

$$X(t) = t_E(t)NPP(t) - X_p(t)$$

Transient dynamics

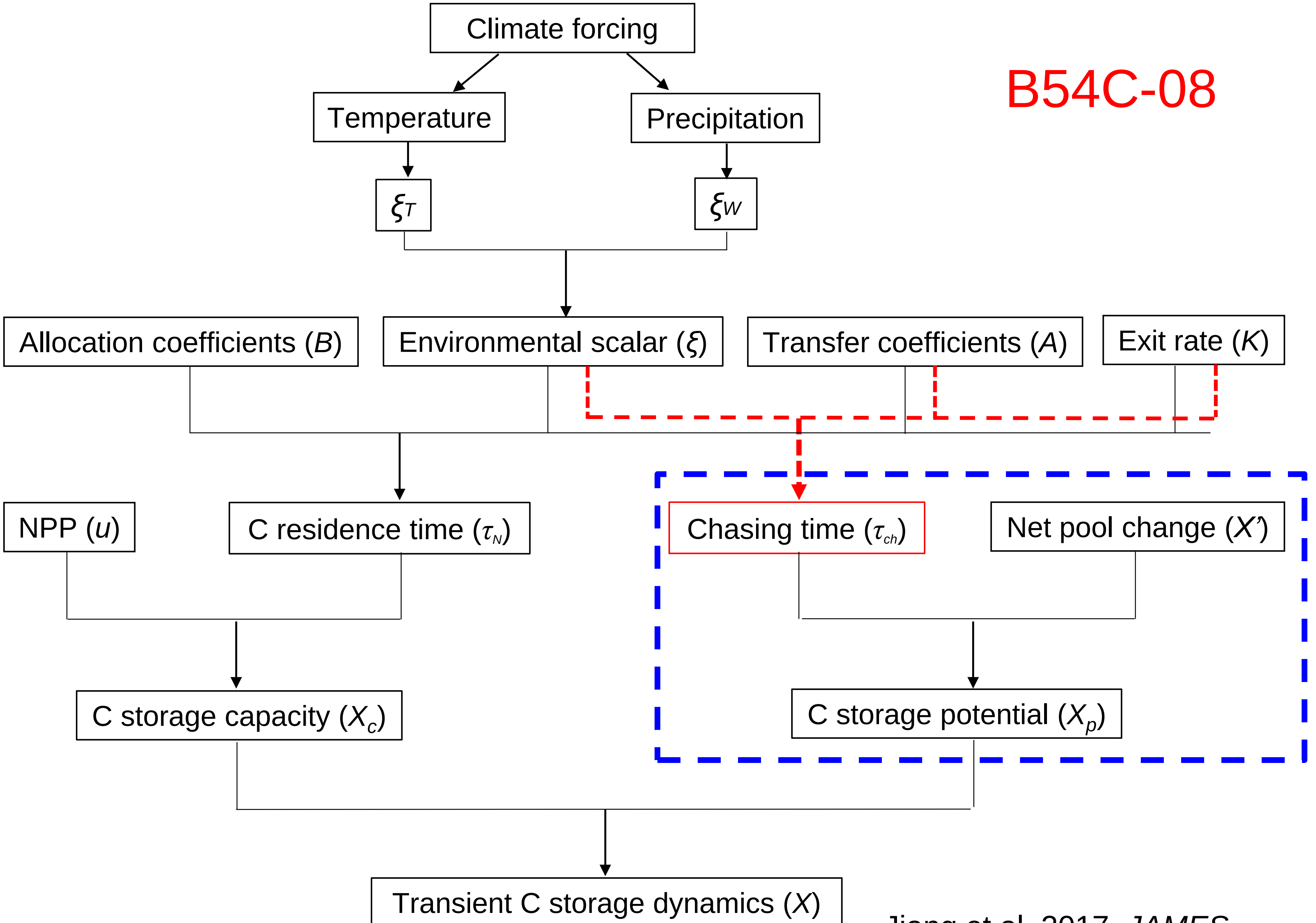
Residence time   Production   Potential

Three dimensions
3D

Luo et al. 2017, *Biogeosciences*

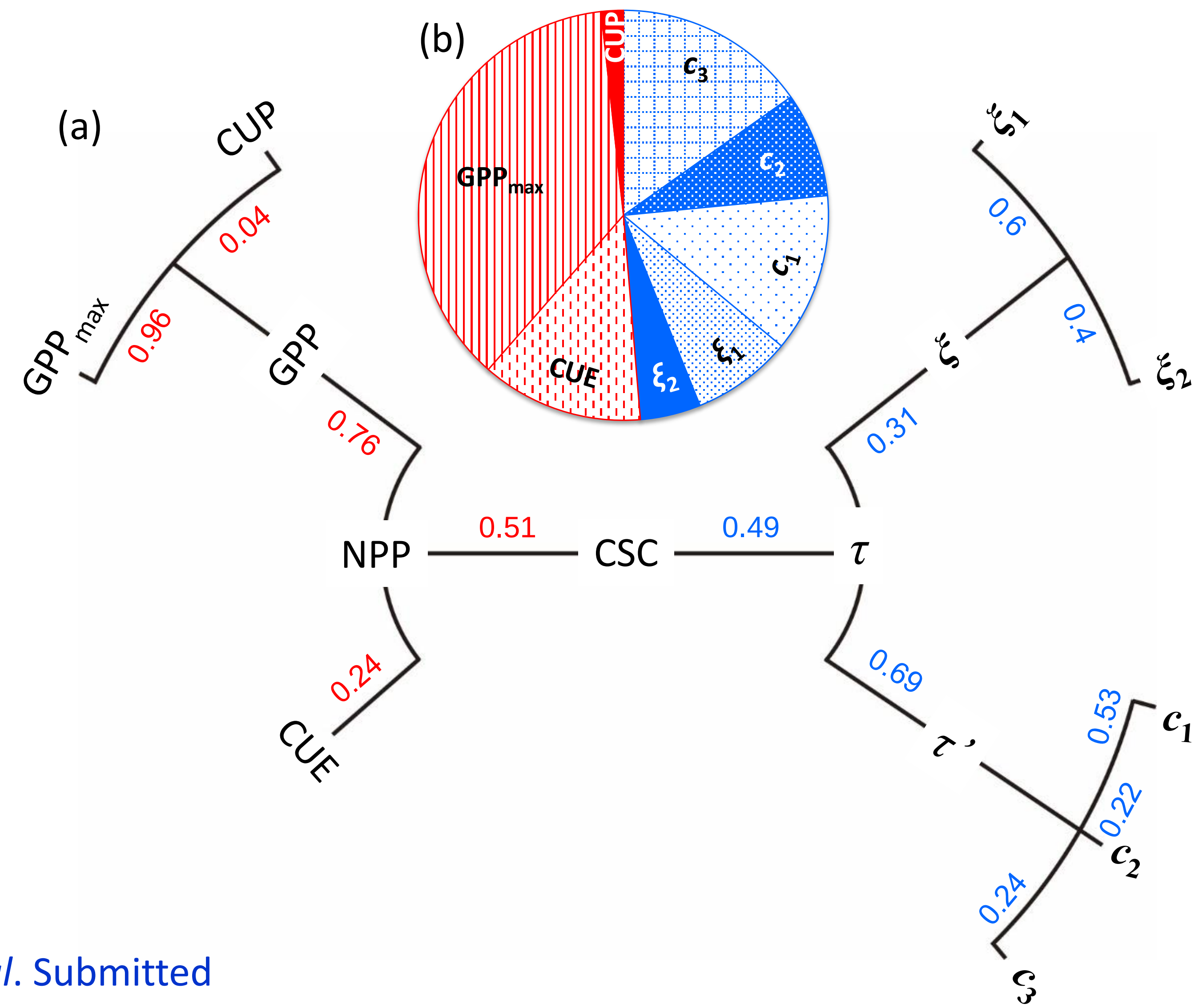
Transient Traceability Framework (TTF)

B54C-08



Jiang et al. 2017, *JAMES*

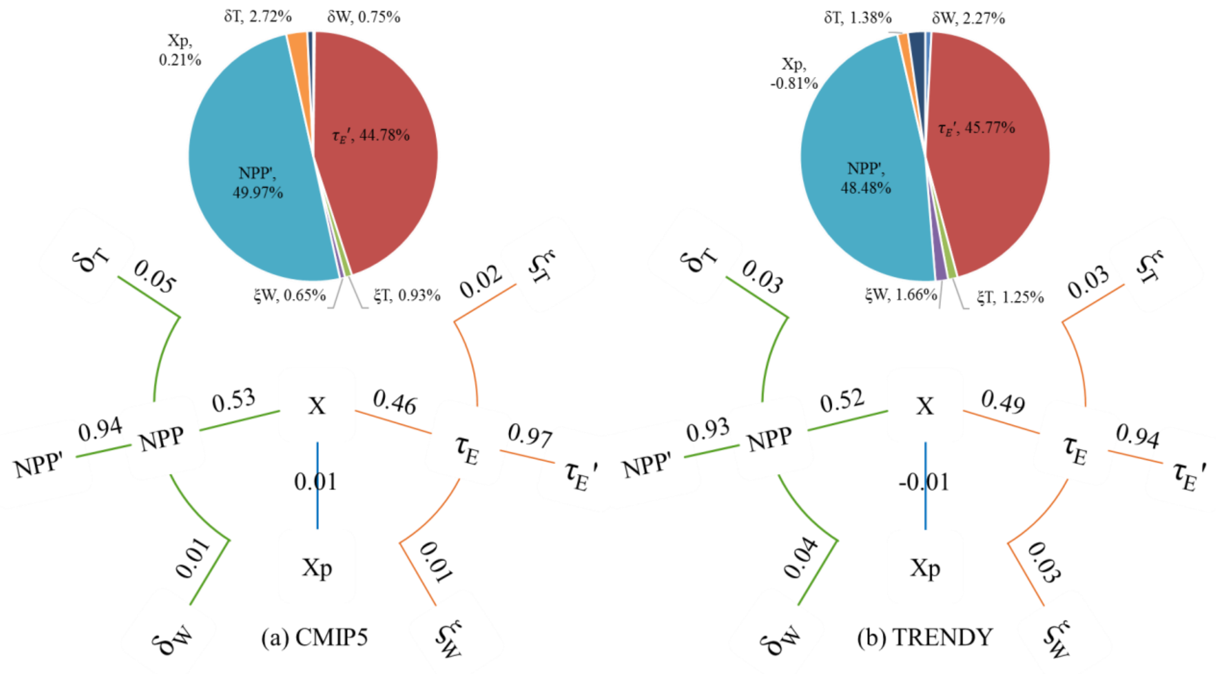
FACE Data-Model Synthesis

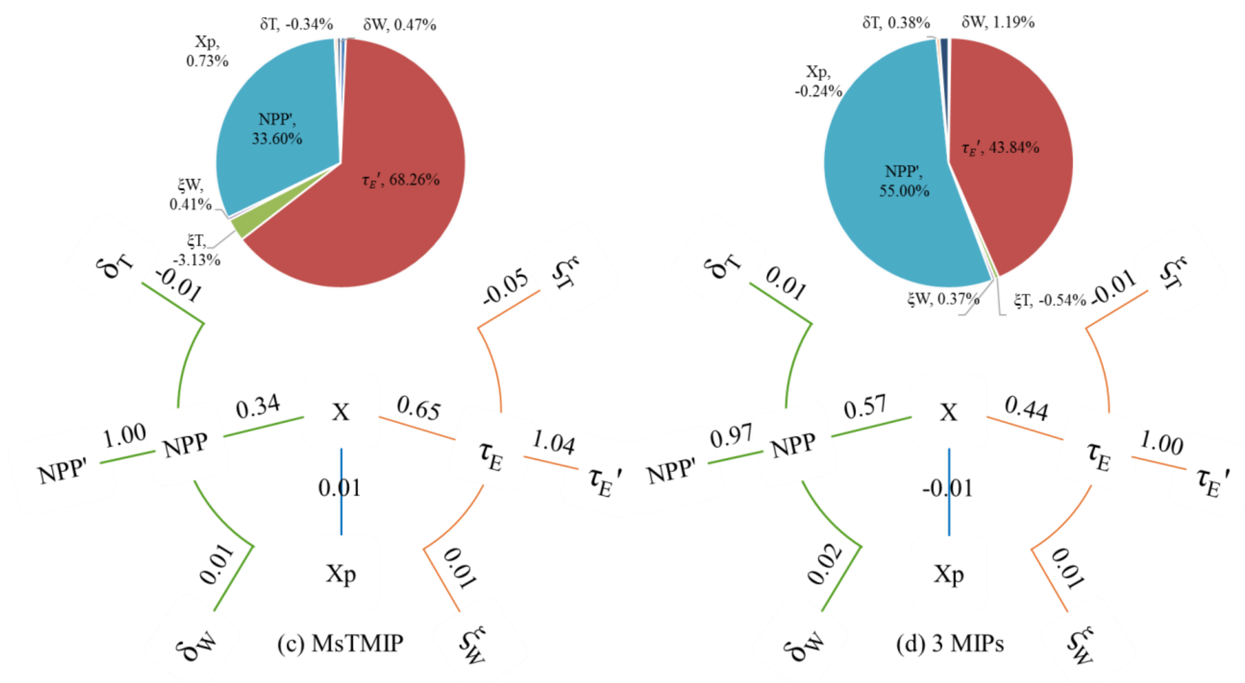


Xia *et al*. Submitted

Three MIPS

CMIP5
TRENDY
MsTMIP



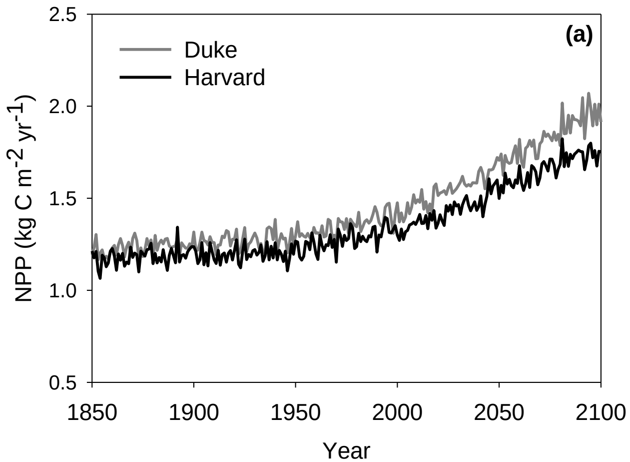


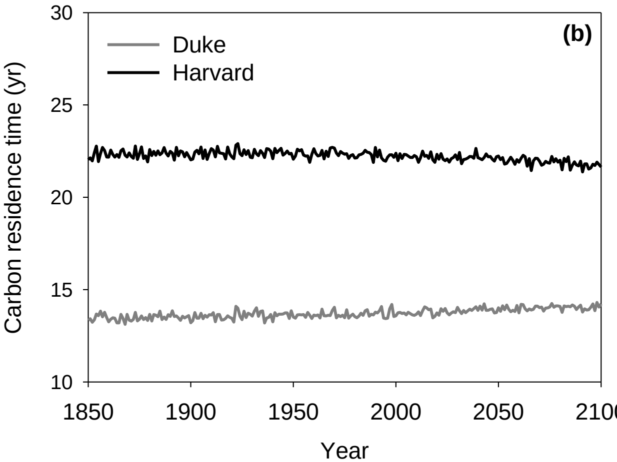
Zhou et al. 2018
J Climate

Ecosystem responses to climate change

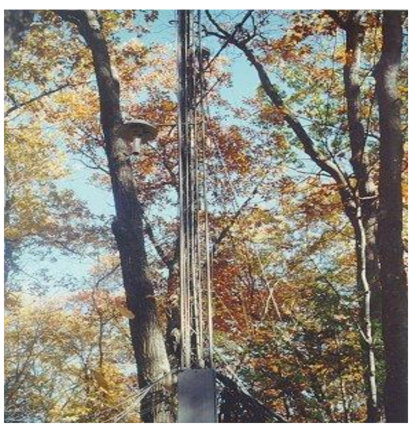
B54C-08

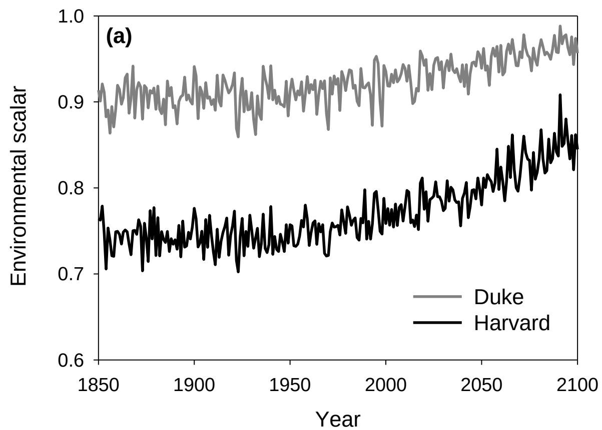
Duke Forest

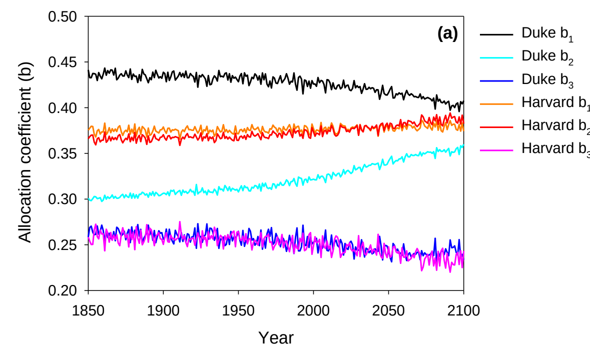




Harvard Forest







Jiang et al. 2018, *JAMES*

1-3-5 scheme for uncertainty analysis

- One (1) formulae unifies all land C cycle models
- One 3-D space (input, residence time, and sink potential) to evaluate all model outputs
- Five (5) Traceable components to pinpoint uncertainty sources down to individual line of code or values of parameters

Other benefits

- Most likely make your life easier
  - Simplicity in coding
  - Cleaner and more efficient code
  - Faster for spin-up
- Enabling new research
  - Sensitivity analysis (e.g., Sobol)
  - Pool-based data assimilation
  - Diagnostic variables (e.g., residence times)
  - Traceability of uncertainty sources
- Understanding your model results much easier